AN FE MODEL TO INVESTIGATE HIGH SPEED CRACK PROPAGATION

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M. SIVA REDDY

to the
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JUNE, 1997

CERTIFICATE

It is certify that the work contained in the thesis entitled **"AN FE MODEL TO INVESTIGATE HIGH SPEED CRACK PROPAGATION"**,by *M. Siva Reddy,* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Dedicated to

MY PARENTS

CONTENTS

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ABSTRACT

Plate problems often involve dynamic disturbances by time dependent external forces or displacements. The purpose of this study is to investigate the effect of dynamic loading on propagation of cracks in a plate. As it is not possible to obtain closed form solutions for dynamic fracture mechanics problems of practical interest, one has to make use of numerical method.

The existing force release models to simulate crack propagation have certain drawbacks giving raise to large oscillations in the solution. In the present work a new model is developed to study high speed propagation of cracks. In this model instead of reducing the crack tip node force, an additional one-dimensional elastic element is attached to the crack node, and the stiffness value is gradually reduced as the crack proceeds to next node.

The FE code is first validated with a static problem and after validation it was used to solve quasistatic and dynamic crack propagation problems and then extended to solve the inverse problem. The energy release rate due to the crack propagation with respect to time is calculated for different propagation speeds and found to be more consistent with practical results.

LIST OF SYMBOLS

a	Crack length
A	Area of crack
B	Width of specimen
[B]	Set of dervitave of interpolation functions
c	Dilatation wave velocity
d	Distance between two nodes
[D]	Elastic costitutive relation
E	Youngs modulus
F_{HB}	Holding back force at crack tip
[F]	Global force vector
G	Energy release rate
J	Fracture toughness
K_s	Stiffness of spring
[K]	Global stiffness matrix
[M]	Global mass matrix
[N]	Set of interpoletion functions
π	Potential energy
ρ	Density
ν	Poissions ratio
[t]	Traction vector
T	kinetic energy
U	Strain energy
[U]	Displacement vector
v	Volume
W_{ext}	External work done

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION:

The dynamic fracture phenomenon can be characterized by various dynamic states of crack tip. The dynamic states of crack tip are induced by impact loading applied to the cracked solids, or by fast motions of the crack tip itself. Dynamic loads may be created by moving vehicles, wind, gusts, shocks or blasts, unbalanced machines, wave impacts, seismic disturbances etc. For solving these problems often the time variation is neglected and they are treated as static or quasistatic problems. Structural members which are subjected to cyclic loading such as vibrations, microscopic imperfections in the material give rise to microscopic cracks which in turn grow into cracks of significant size over passage of time. At this stage further growth of cracks stably or unstably leading to the component failure depends not only on the material characterising but also on the nature of loading, i.e static or dynamic. For many dynamic problems it is impossible to find closed form solutions, especially if they involve the aspect of fracture. A number of analytical solutions to problems of dynamic fracture shedding important light on the basic phenomenon have been obtained in the past two decades. These solutions are however limited to simple cases of loadings and of unbounded plane bodies. Usually, the interactions of the stress waves emanating from the crack tip and or those reflected from the boundaries of a finite body make the analytical solutions invalid. Thus, it often becomes mandatory to analyze dynamic crack propagation in finite solids using numerical methods.

The subject of the mechanics of dynamic fracture can be broadly described as that branch of mechanics of solids, where in the effect of material inertia and stress waves on crack play significant roles. One method of classifying dynamic fracture problems is as follows [Nishoka and Atulri, 1986]:

(i) Solids containing stationary cracks subjected to dynamic loading

(2) Solids containing dynamically propagating cracks under quasi-static loading

(3) Solids containing dynamically propagating cracks under dynamic loading

A knowledge of time-dependent asymptotic stress and displacement fields near the crack tip, and their "strength" as quantified, for instance, by the "stress intensity factor (SIF) in linearly elasto-dynamics, is basic in understanding the process and nature of dynamic fracture in solids. A direct approach to laboratory evaluation of dynamic fracture toughness and crack arrest toughness, is based on the laboratory measurements of the instruments dynamic stress fields close to the propagating crack tip using photo elastic method. Such direct measurements, however, are generally difficult. To overcome these difficulties, and simplify the measurements necessary, hybrid experimental-numerical methods are often preferred. This is one reason that the advancement of the state of science of dynamic fracture mechanics relies heavily on simultaneous advances in computational methods.

Dynamic fracture mechanics concerns (i) the onset of crack growth under dynamic loading (ii) dynamic crack propagation in stressed solids. The conventional linear elasto-plastic (or) quasistatic fracture mechanics applies only up to the end of stable growth under quasistatic loadings and assumes that the onset of unstable

crack propagation renders the structure useless. Yet, the prevention of crack growth initiation itself may be too conservative or too costly an objective for some structural designs, and also catastrophic failures caused by unstable crack growth are obviously intolerable. In these cases, the assurance of crack arrest, as a second line of defense, is essential. This concept has been investigated for design methodologies assuring the integrity of nuclear pressure vessels under thermal shock conditions, LNG ship hulls, and gas transmission pipelines.

The fundamental frame work of the subject of dynamic fracture mechanics relies primarily on solutions for dynamic behaviour of solid containing cracks these solutions are characterized by (i) stress wave integrations (ii) the need to account for kinetic energy in the global energy balance of the fracturing body and (iii) inertia effects of the material. The inertia effect may arise due to two reasons (i) rapidly applied load on a cracked solid and (ii) rapid crack propagation. In the first case the influence of the load is transferred to the crack by stress waves through the material. In the second case the material particles on the two crack faces displace with respect to each other, as the crack advances. The inertia effect, in the first case, is considered significant when the time taken to load the specimen to maximum value is small as compared to the time required for a characteristic dimension of the body. In the second case, the inertia effect should be accounted far whenever the crack velocity is a significant fraction of the characteristic velocity.

In the past two decades or so, a number of analytical solutions, which provides a useful understanding of dynamic crack behaviour, have been obtained. These analytical solutions are however, limited to cases of simple loadings and unbounded plane

bodies. Moreover the stress wave introductions, which play an important role in dynamic fracture mechanics. Usually render the analytical solutions intractable. Therefore, the use of numerical methods is often indispensable for the analysis of cracks in finite solids.

Numerical methods in fracture mechanics were initially appraised in 1978 at that time, a comparison of the finite element and finite difference methods to the following conclusions. The finite element method was more suitable, for the analysis of stationary cracks under dynamic loadings. Due to the fact that the relevant singularities can be modeled in the crack tip elements. In large measure, this is the result of the development of finite element procedures to model propagating singularities and of path-independent integrals which characterize the strength of the fields near propagating crack tip.

1.2 LITERATURESURVEY:

Though considerable amount of work have been done to study the fracture phenomena through numerical methods, only large rectangular double cantilever beam specimens were given special interest Owen and Shantaram (1977) started the case of finite element method to study dynamic crack growth. They studied double cantilever beam specimen and pipeline problems under transient loading. The crack was advanced from one node to the next node, when the stress at the gauss point nearest to the crack tip exceeded a certain value and then damping coefficient was made zero at the released node. The crack propagation history was simulated for the given loading conditions.

Nishoka and Atluri (1982a) investigated the crack propagation and arrest in a high strength steel DCB specimen using moving singular dynamic finite element procedure. An edge crack in a rectangular DCB specimen was propagated by inserting a wedge. The results were compared with experimental data of caustics. In another work, Nishoka and Atluri (1982b) presented the results of generation and prediction studies of dynamic crack propagation in plane stress and plane strain cases. The studies were conducted by using FEM, taking into account stress singularity near the crack tip. The variation of dynamic stress intensity factor with time and the variation of dynamic fracture toughness with velocity were studied and compared with available experimental results.

Nishoka and Atluri (1986) gave elaborate information on analysis of dynamic fracture using FEM. The most common ways to deal with the crack tip region are to simulate crack growth through gradual release of element nodal forces or embedding a moving element in which the interpolation functions are determined by the continuum near tip fields at the crack tip in the mesh, and J-Integral considerations. Following spacial description the differential equations in time for the node point variables must be integrated. Because the dynamic fields associated with rapid crack growth are rich in high frequency content, small time steps are required for accuracy. Because of natural restriction to small time steps, the authors report that many times combination of conditionally stable explicit time integration scheme and diognalized mass matrix are found to be accurate.

Cronech and Williams (1987) used dynamic and generation mode finite element program to analyze different specimen geometries. Chiang (1990) presented a numerical

procedure based on eigen functions to determine the dynamic stress intensity factor of crack moving at steady state under antiplane strain condition. An edge crack problem and a radial crack problem was solved using traction and displacement boundary conditions separately. It was shown that the dynamic effect is relatively insignificant for low crack propagation speeds provided that specimen size fairly is large.

A detailed analysis of the various time integration schemes from the point of view of accuracy and speed, were reported by (Scran et al., 1990; Zienkiewicz and Taylor, 1991, Bathe, 1990). It was concluded that, even though central differences scheme is the best in terms of computational speed and cost, Newmark's constant average scheme gives better accuracy and offers unconditional stability, whereas, the time step size has negligible effects when central difference time integration scheme is employed (Bazant and Celep, 1982, 1983).

Thesken and Gudmundson (1991) worked with an elasto-dynamic moving element formulation incorporating a variable order singular element to enhance the local crack tip description. Kennedy and Kim (1993) incorporated micropolar elasticity theory into a plain strain finite element formulation to analyze the dynamic response of the crack. Material with strong micropolar properties were found to have significantly lower dynamic energy release rate than their classic material counter-parts, Wang and Williams (1994) investigated high speed crack growth in a thin double cantilever beam specimen using FEM. The cantilever end was loaded under an initial step displacement of 1 mm and then pulled with a constant velocity the crack was propagated at assumed speed by gradual node release technique. Large dynamic effects were observed because of wave reflections within

finite specimen size. The reflection and interaction of the waves from the free boundary were avoided by limiting the duration of study.

1.3 PRESENT WORK:

In the present work a new model is developed to study highspeed propagation of cracks in a 2-dimensional plate, for simplicity the crack is symmetric and is under modeI propagation. In this model instead of reducing cracktip node force, an additional one-dimensional elastic element is attached to the crack node , whose stiffness value is gradually reduced to zero as the crack proceeds to next node. This model is expected to avoid the oscillations in solution. The present work makes use of some parts of the code developed for the dynamic analysis of two dimensional problem by Kishore, Kumar and Verma [1995].

Chapter 2 deals with the finite element formulation and evaluation of fracture parameter and explains the details of crack propagation models. Chapter 3 discusses the new model developed in the present analysis. Chapter 4 deals with the validation of code and results for the test problems. Chapter 5 deals with conclusions of the present work and the scope of further work.

CHAPTER2

FEM FORMULATION AND CRACK MODELLING

2.1 FINITE ELEMENT FORMULATION:

The hyperbolic partial differential equation governing elastic wave propagation in homogeneous,. anisotropic and linear elastic solid is,

$$C_{ijkl}u_{k,lj} = \rho\ddot{u}_i \tag{2.1}$$

The total potential of a linear elastic body, neglecting body forces and using D'Alembert's principle, is given by

$$\Pi = \int_v u_{i,j}\tau_{i,j}\,dv - \int_s u_i t_i\,ds + \int_v u_i\rho\ddot{u}_i\,dv \tag{2.2}$$

where the first term is the strain energy, second term is the workdone by the surface traction t_i and the last term is the work done by the inertia forces. The integrations are carried out over the volume of the body v or surface s, whichever is applicable.

The finite element solution involves the discretization of the domain into suitable elements, approximation of the field values interior to the elements in terms of its nodal values through the shape functions of the chosen elements and the determination of the nodal values through the minimization of the above energy

functional. Invoking the stationarity of π, namely, $\delta\pi = 0$, the following equation may be obtained

$$\int_v \delta u_{i,j}\tau_{i,j}\, dv - \int_s \delta u_i t_i\, ds + \int_v \delta u_i \rho \ddot{u}_i\, dv = 0 \tag{2.3}$$

Approximating the field values (u) within an element in terms of its nodal values (U) as

$$u^e(x,y,z,t) = N^e(x,y,z)U^{ne}(t) \tag{2.4}$$

From this following strain-displacement relation is obtained

$$\epsilon^e(x,y,z,t) = B^e(x,y,z)U^{ne}(t) \tag{2.5}$$

The constitutive equation is

$$\tau^e(x,y,z,t) = D^e\ \epsilon^e(x,y,z,t) \tag{2.6}$$

N^e is the displacement interpolation matrix, B^e is the strain-displacement matrix and D^e is the material property matrix.

Substitution of Eqs. (2.4) to (2.6) into (2.3) gives the equation of motion in the matrix form as,

$$[M]\left[\frac{\partial^2 U}{\partial t^2}\right] + [K]\ [U] = [F] \tag{2.7}$$

where [M] and [K] are the system global mass, stiffness matrices are obtained by

assemblage of elemental stiffness, mass matrices and [F] is external load vector. [U] is global displacement vector of finite element assemblage.

$$[M] = \Sigma[M]^e,\ [K] = \Sigma[K]^e,\ [F] = \Sigma[F]^e$$

where $[M]^e$, $[K]^e$ and $[F]^e$ are the elemental mass, stiffness and load matrices respectively. In the wave propagation analysis the body is usually descritized using simple elements such as 3 -noded triangular or 4 -noded quadrilateral element to avoid spurious reflections at element boundaries. In the present work 4 -noded element is used. Using the general terminology of the elemental matrices can be briefly explained as follows:

$$[M]^e = \int_{v_e} \rho [N]^T [N] dv \tag{2.8}$$

$$[K]^e = \int_{v_e} [B]^T [D][B] dv \tag{2.9}$$

$$[F]^e = \int_{s_e} [N][t] ds \tag{2.10}$$

where

[N] is set interpolation functions by which one can obtained displacements at any spatial point from nodal values.

[B] is set of derivative of interpolation functions.

[D] is the material matrix.

[t] is the traction vector.

The equation 2.7 can be solved either by time integration or by mode superposition of which the former is preferred in the wave propagation problem. In this integration scheme, there are many different methods, which can be classified as "explicit" or implicit" whose advantages and disadvantages of each method are given in detail in Bathe (1990). In the present work Newmark integration method for time variable is used. The details of the Newmark method are given in Bathe (1990). The choice of time step Δt for time integration is an important factor for an accurate solution. An optimum choice of time step is $\Delta t = d/C_d$ in which d is the smallest element size, and C_d is dilatation wave velocity.

2.2 EVALUATION OF FRACTURE PARAMETERS

The crack effects are usually characterised by stress intensity factors (K_I, K_{II}, K_{III}),energy release rate (G_I, G_{II}, G_{III}) and J-Integral. While the SIF's require special(moving) elements to be used, the other two give reasonably accurate results with ordinary meshes. These two parameters are described in the following sections.

2.2.1 Energy Release Rate

Griffith proposed that a crack in a body will not extend unless energy is released in the process to overcome the energy needs of forming two new surfaces, one below and one above the crack plane. As the crack in the plate advances, the plate becomes less stiff, consequently, for the case of the plate with ends held rigidly, the stress within the plate decreases, and the strain energy stored in the plate is reduced. The energy thus

released is available for the crack to grow. Most of the energy released, as the crack advances comes from those parts of the plate adjacent to the cracked planes. There are two important quantities involved (i) how much energy is released when a crack advances and (ii) minimum energy required for the crack to advances in forming two new surfaces. The first quantity is measured in terms of a parameter energy release rate, denoted by 'G'. Thus the energy release rate is defined as energy release per unit area extension of crack growth if there is a virtual crack growth, i.e, energy equal to G would be released from the system per unit extension of area. The energy requirement for a crack to grow by a unit area is called crack resistance. Crack resistance is sum of two energies required to (i) form new surfaces and (ii) cause anelastic deformation. Both the available energy release rate and crack resistance are important to study the possibility of crack becoming critical. Obviously, the available energy release rate must be greater than crack resistance for a crack to have a chance of grow. If the energy release rate exceeds the crack resistance,the crack starts to grow at high speeds.

Mathematical Formulation:

As the crack advances the following phenomena happen:

1. Energy in the component decreases
2. Stiffness of the component decreases
3. Energy is consumed to crack two new surfaces.

Formulation for energy release rate is carried out invoking the principle of conservation of energy. First, consider quasistatic crack growth case. Let the incremental increase

in the crack area be ΔA. This crack growth is achieved by an incremental external work, ΔW_{ext} done by the external forces and the strain energy within the body of the component increases by ΔU then available energy, $G \Delta A$ provides the energy balance as follows:

$$G\Delta A = \Delta W_{ext} - \Delta U \tag{2.11}$$

$$G = \frac{-d}{dA}(U - W_{ext}) \tag{2.12}$$

$$G = \frac{-d\pi}{dA} \tag{2.13}$$

where π is potential energy

If the plate of uniform thickness B then ΔA can be expressed as B da.

da is increment in crack length.

In dynamic case, as crack moves rapidly some energy is being consumed to impart kinetic energy to cracked portion of the body and to generate stress waves. Therefore G in the dynamic case can be far different from quasistatic case. Thus Eq.3.1 can be written as

$$G\Delta A = \Delta W_{ext} - \Delta U - \Delta T \tag{2.14}$$

where ΔT is increment in kinetic energy in the body

$$G = \frac{1}{B}\left(\frac{dW_{ext}}{da} - \frac{dU}{da} - \frac{dT}{da}\right) \tag{2.15}$$

in terms of FE analysis the above terms can be written as

$$W_{ext} = [U]^T[R]$$

$$U = \frac{1}{2}[U]^T[K][U]$$

$$T = \frac{1}{2}[\dot{U}]^T[M][\dot{U}]$$

where

W_{ext} = external work

U = strain energy

T = kinetic energy

[R] = external load vector

[U] = displacement vector

[$\acute{U}$] = velocity vector

[K] = Stiffness matrix

[M] = mass matrix

2.2.2 J-integral:

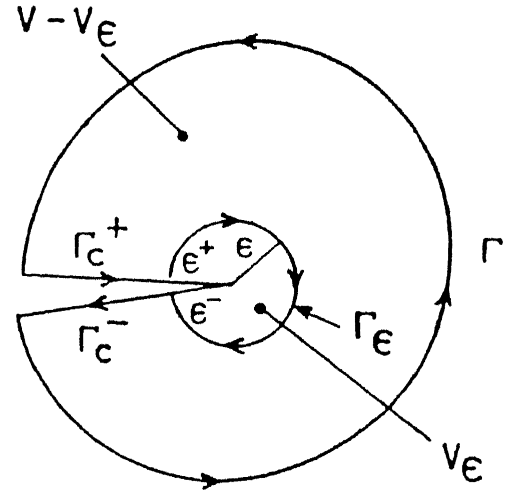
Under approximate assumptions of material homogeneity, the strength of the crack tip field is governed by an integral evaluated over a path that is far removed from the crack tip. Since the stress and displacement data are evaluated away from the crack tip, these values are relatively insensitive to the linear details of modelling of crack tip region. J-integral is used for computing elasto-static fracture is mode I. Also the far

field integral can be calculated with reasonable accuracy using relatively course finite element models of the structure.

Atluri (1982) and Nishioka and Atluri (1983) have under taken the studies of several path independent integrals in the analysis of crack growth initiation and propagation of cracks in elastic or inelastic materials under quasistatic and dynamic conditions. In the present work J is defined as follows

$$J = \lim_{\varepsilon \to 0} \int_{\Gamma+\Gamma_c} \left[Wn_1 - t_i u_{i,1}\right] dS + \int_{V-V_\varepsilon} \rho \ddot{u}_i \, u_{i,1} \, dV \tag{2.16}$$

where W is strain energy density, n_1 is the direction of unit outward normal, t_i is the surface traction and definition of the paths Γ_ϵ, Γ, Γ_c and the volume V, V_ϵ are shown in Fig. 2.1. The value of $\hat{J}$ is independent of the choice of Γ of Fig. 2.1 only under steady state crack growth conditions. The path is held stationary as the crack tip is extended in a self similar manner.



$$\Gamma_c = \Gamma_c^+ + \Gamma_c^-$$

$$\delta(V - V_\epsilon) = \Gamma + \Gamma_c - \Gamma_\epsilon$$

Fig. 2.1 Contour of J-integral

2.3 MODELLING OF CRACK PROPAGATION:

To simulate the crack propagation in solids two different concepts of computational modelling have been in use, ie. the stationary mesh procedure and moving mesh procedure. Out of these two moving mesh procedure is the name implies, involves the changing of mesh for each iteration as the crack propagation. This is computationally very intensive. Stationary mesh procedure does not require change in the mesh in general, and requires only change in boundary and loading conditions. This procedure is explained in detail in the following paragraphs.

Stationary mesh procedure:

In the "stationary mesh" procedure of modelling linear elastodynamic crack propagation, when the node spacing is $c\Delta t$ (c being crack velocity), one only has to release the constraint on displacement at the old crack tip. If one also applies equal and opposite nodal forces to eliminate reaction forces at the old crack tip, the crack surface becomes over relaxed.

As the crack propagation velocity is lower than the wave velocities, in general, the crack-tip occupies positions in between the nodes at various time step of the numerical integration. Thus if a simple i.e.,crack propagation procedure by advancing tip from node to next node is used, then the crack-tip either stay at one node or jump from one node to next node in time Δt in the computer simulation. This causes the crack tip movement to be irregular or discontinuous, serious violations of the kinematics

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and fraction condition ahead of the actual crack-tip will result. Also sudden increase of crack length by the release of constraints on displacement induces spurious high frequency oscillations in the finite element solutions. To overcome such difficulties, several algorithms have been suggested in literature to release the node gradually. Suppose that the actual tip is located at'C' is between the finite element nodes B and D as shown in Fig. 2.2. The lengths segments BC and BD are b and d respectively. The holding back force, F, at node b is gradually reduced to zero as the crack tip reaches to the node D. The various scheme available to decrease the force to zero as follows :

(i) Malluck and King (1978) suggested the release rate based on constant stress intensity factor; that is

$$\frac{F}{F_o} = \left(1-\frac{b}{d}\right)^{1/2} \tag{2.17}$$

Where F_0 is original reaction force when the crack tip was located at node B.

(ii) Pydham et.al (1978) suggested the release rate based on constant energy release rate.

$$\frac{F}{F_o} = \left(1-\frac{b}{d}\right)^{3/2} \tag{2.18}$$

(iii) Kobayasi et.al (1978) suggested the linear release rate based on no physical argument other than pure intuition.

$$\frac{F}{F_o} = \left(1-\frac{b}{d}\right) \tag{2.19}$$

For having gradual and smooth opening of crack Kishore, Kumar and Verma (1993) tried an alternative method. The holding back force at the crack tip B is linearly decreased to zero when the crack reaches the end of the next element. Thus, when the crack tip goes beyond node B then

$$\frac{F_B}{F_{HB}} = \left(1 - \frac{b}{d+b_1}\right) \tag{2.20}$$

Where F_{HB} is reaction at node B, when the node was closed; b is the crack extension and d, d_1 are the element lengths as shown in Fig. 2.2 and when crack moves beyond node D to point D_1

$$\frac{F_B}{F_{HB}} = \left(1 - \frac{d+b_2}{d+d_1}\right) \tag{2.21}$$

$$\frac{F_D}{F_{HD}} = (1 - \frac{b_2}{d_1+d_2}) \tag{2.22}$$

Where F_{HD} is the reaction at node D, when the nodes was closed; b, b_2 are the crack extensions and d, d_1, and d_2 are the element lengths as shown in Fig. 2.2.

In the present work instead of reducing the force a one-dimensional elastic element is attached to the crack node whose stiffness is reduced gradually as the tip advances to the next node. This model explained in detail in Chapter3.

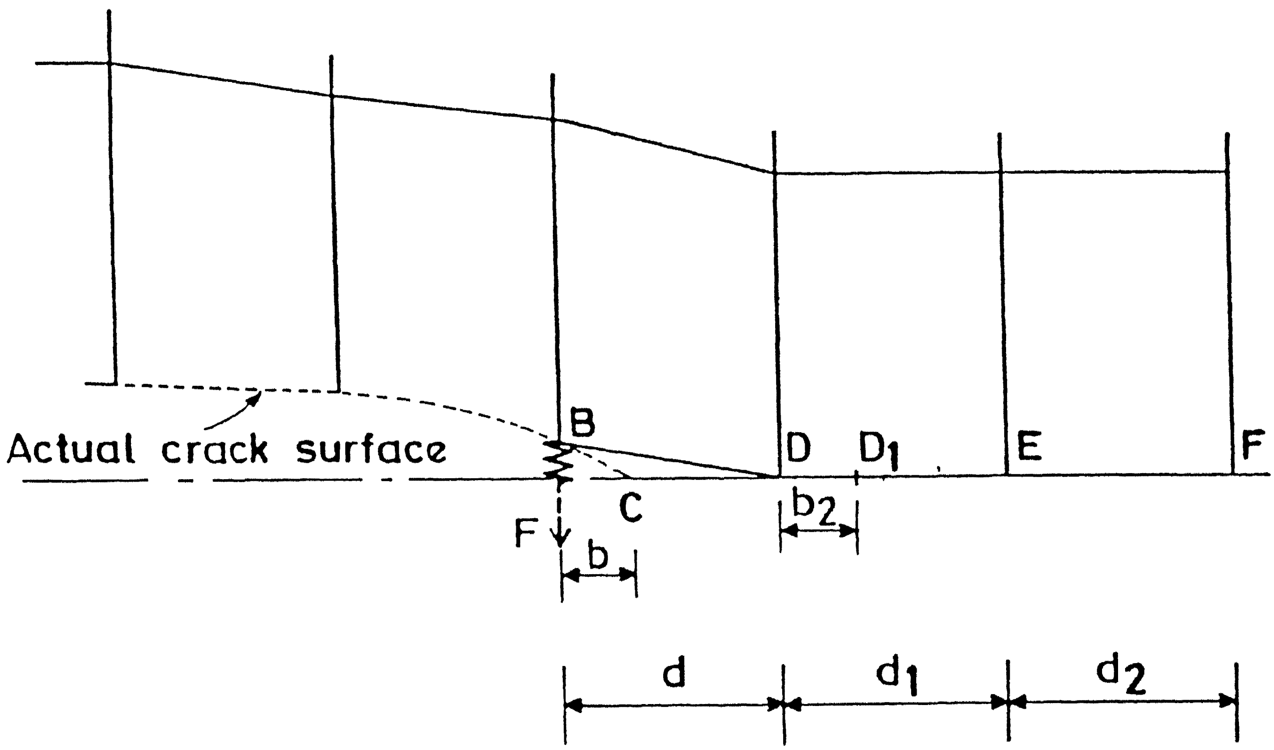

Fig. 2.2 Crack opening scheme in force release model

CHAPTER 3

STIFNESS RELEASE MODEL

3.1 INTRODUCTION

In the previous Chapter force release models to simulate crack propagation were described. These models when applied to high speed crack propagation have the following drawbacks:

(1) The time increment Δt, is usually taken such that it takes 15 to 20 iterations for the cracktip to cross one element. The amount of force necessary to keep the nodes together is determined and a factor is used to proportionately decrease this force as the crack advances. Thus, at subsequent times, though the problem is highly dynamic, the current calculations were based on the force calculated to keep the nodes closed 15 to 20 time steps before. This will cause discripancy as the crack advances further from one element to the next one causing large oscillations in solution.

(2) In the case of decreasing energy release rate and possible crack arrest in between the nodes, freezing the force at particular value in the dynamic problem will lead to a non acceptable solution. This aspect acquires importance in the case of inverse problems.

In the present work to avoid the above difficulties a modified model has been developed.

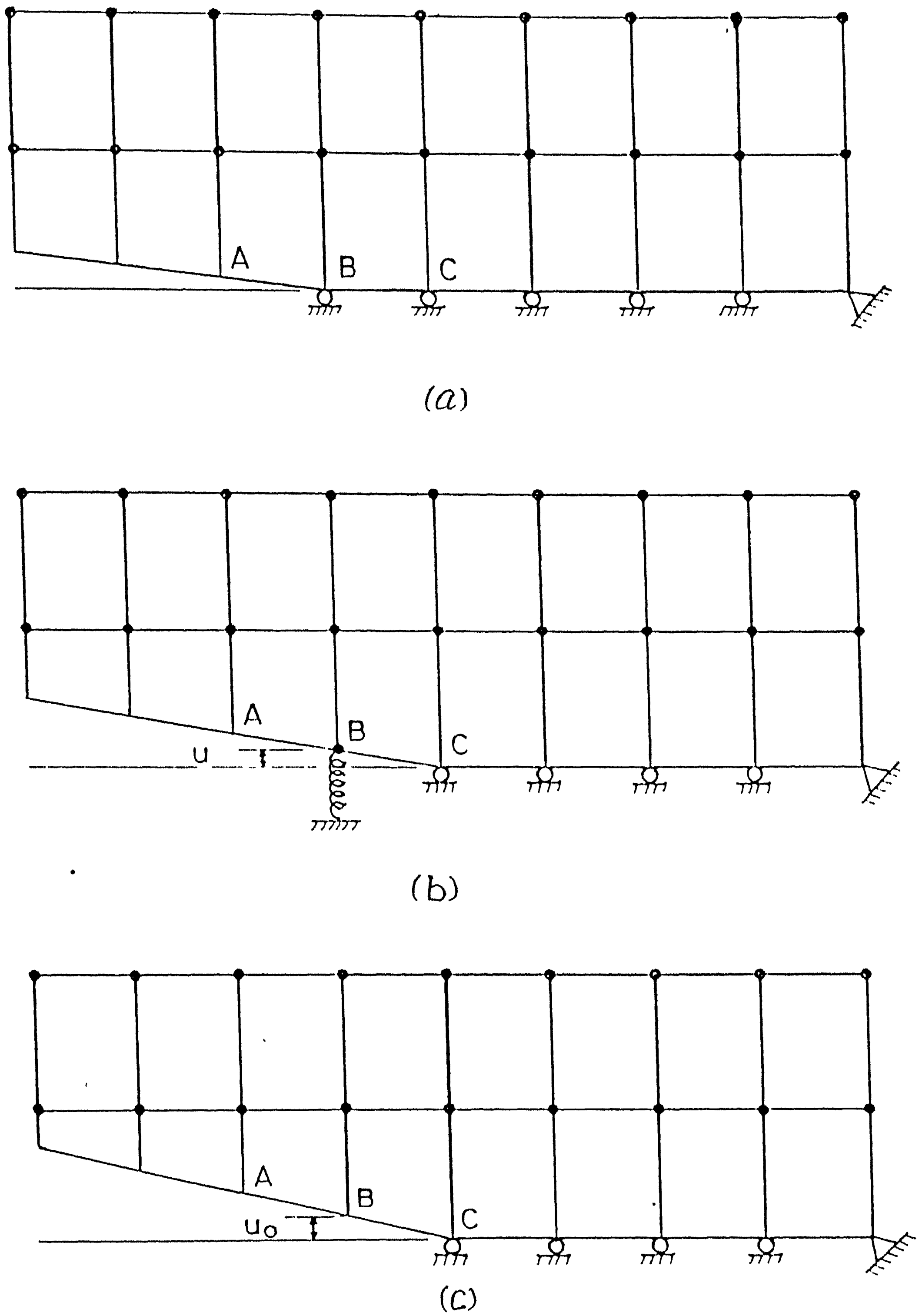

Fig. 3.1 Crack opening scheme in stiffness release model

3.2 STIFFNESS RELEASE MODEL

In this model instead of applying a force at the opening node a one dimensional elastic element is added (Fig. 3.1) whose stiffness is reduced gradually as the crack tip advances to the next node. This additional stiffness lets the crack to be opened in a controlled manner. It is clear that the amount of additional stiffness required is very large (theoretically infinite) when the crack tip is at the begining of element and zero when the crack tip reaches the other end. An expression for additional stiffness K_S to be added is derived with reference to quasistatic crack propagation.

Fig. 3.1(a) shows a symmetric part of a deformed plate in which crack is upto the node B. Fig. 3.1(c) shows the configuration of the plate when the crack has completely propagated to the next node C. Fig. 3.1(b) shows the modelling of the plate with partially propagated crack (somewhere between B and C) by the addition of a one-dimensional linear spring element whose stiffness is K_S. Let K_0 be the stiffness of the node B of the original plate without additional spring (i.e., force required to cause unit displacement of node B), u_0 be the displacement of the node when the crack tip reaches the next node and the element completely opens. Then, K_S can be found from the equilibrium equation:

$$K_o(u_o - u) = K_s\, u \tag{3.1}$$

$$K_s = K_o \left(\frac{u_o}{u} - 1 \right) \tag{3.2}$$

Now to express K_S as function of crack size, it is necessary to know 'u'as a function of crack size. For this purpose energy release rate is used. At any stage, the crack can be extended by a small amount by reducing K_S by an amount ΔK_S. This reduction in the spring stiffness causes a decrease in the total energy of the plate. As this is the only energy loss in the system, this can be directly used while evaluating energy release rate.

The energy in the spring can be written as

$$E = \frac{1}{2} K_s u^2 = \frac{1}{2} K_0 \left(\frac{u_0}{u} - 1 \right) u^2 \tag{3.3}$$

The energy release rate is written as,

$$G = \frac{\Delta E}{\Delta A} \tag{3.4}$$

where ΔA is increment in crack area.

Let a non-dimensionalised parameter for crack length, 'a' be defined as,

a = crack length (within the element) / d

where d is the distance between two nodes.

Therefore ΔA can be written as,

$$\Delta A = B \, d \, \Delta a$$

where B is the thickness of plate.

Then energy release rate, G becomes

$$G = \frac{1}{2} K_0 (-u_0) \frac{1}{Bd} \frac{du}{da} \tag{3.5}$$

To determine the variation of u as a function of 'a' it is necessary to know the variation of G. As it can be assumed that within the element G is linear function of crack length, let it be assumed that

$$G = C_1 + C_2 a \tag{3.6}$$

Where C_1 and C_2 are constants to be determined later.

Substituting G in the Eq. (3.5)

$$C_1 + C_2 a = -\frac{1}{2} \frac{K_0 u_0}{Bd} \left(\frac{du}{da}\right) \tag{3.7}$$

$$(C_1 + C_2 a)\, Bd\, da = -\frac{1}{2} K_0 u_0\, du \tag{3.8}$$

Integrating on both sides

$$Bd \left(C_1 a + \frac{C_2 a^2}{2} + C_0\right) = -\frac{1}{2} K_0 u_0 u \tag{3.9}$$

where C_0 is integration constant

$$u = -\frac{2}{K_0 u_0} Bd \left(C_1 a + \frac{C_2 a^2}{2} + C_0\right) \tag{3.10}$$

By applying initial condition u = 0 when a = 0, it can be seen that $C_0 = 0$

using the other end condition $u = u_0$ when a = 1 in Eq. (3.10) it can be obtained.

$$u_0 = -\frac{2}{k_0 u_0} Bd \left(C_1 + \frac{C_2}{2} \right) \tag{3.11}$$

From Eq.(3.10) and Eq. (3.11) it can be seen

$$\frac{u}{u_0} = \frac{2C_1 a + C_2 a^2}{2C_1 + C_2} \tag{3.12}$$

Eq. (3.12) can be simplified as

$$\frac{u}{u_0} = a + C\,(a - a^2) \tag{3.13}$$

where

$$C = -\left(\frac{C_2}{2C_1 + C_2} \right)$$

Therefore, finally Eq. (3.2) can be written as,

$$K_s = K_o \left(\frac{1 - f}{f} \right) \tag{3.14}$$

where $f = u / u_0$ given by Eq. (3.13)

As can be seen that K_0 has to be determined for the given plate under quasistatic conditions and given boundary conditions. However, it may be inferred that the near boundaries affect the value of K_0 than the far off boundaries. Suitable value of 'C' can be determined by trial and error to obtain continuous energy release rate in between elements. Usually it is a very small value. In the present 2-D problem it is of the order of 0.05.

3.3 PREDICTION OF CRACK INITIATION, PROPAGATION AND ARREST:

The present method can be easily applied to predict the crack propagation history under the known loads, if crack propagation resistance of the material is known. That means that fracture resistance of the material, G as a function crack propagation velocity is supposed to be a known input data (Fig. 3.2). At each time step the avaliable energy release rate is determined and it is compared with the material fracture resistant data to predict the crack propagation velocity. This is used to determined the crack propagation length in the ~~the~~ given time interval and propagated accordingly. This procedure continues till crack arrest.

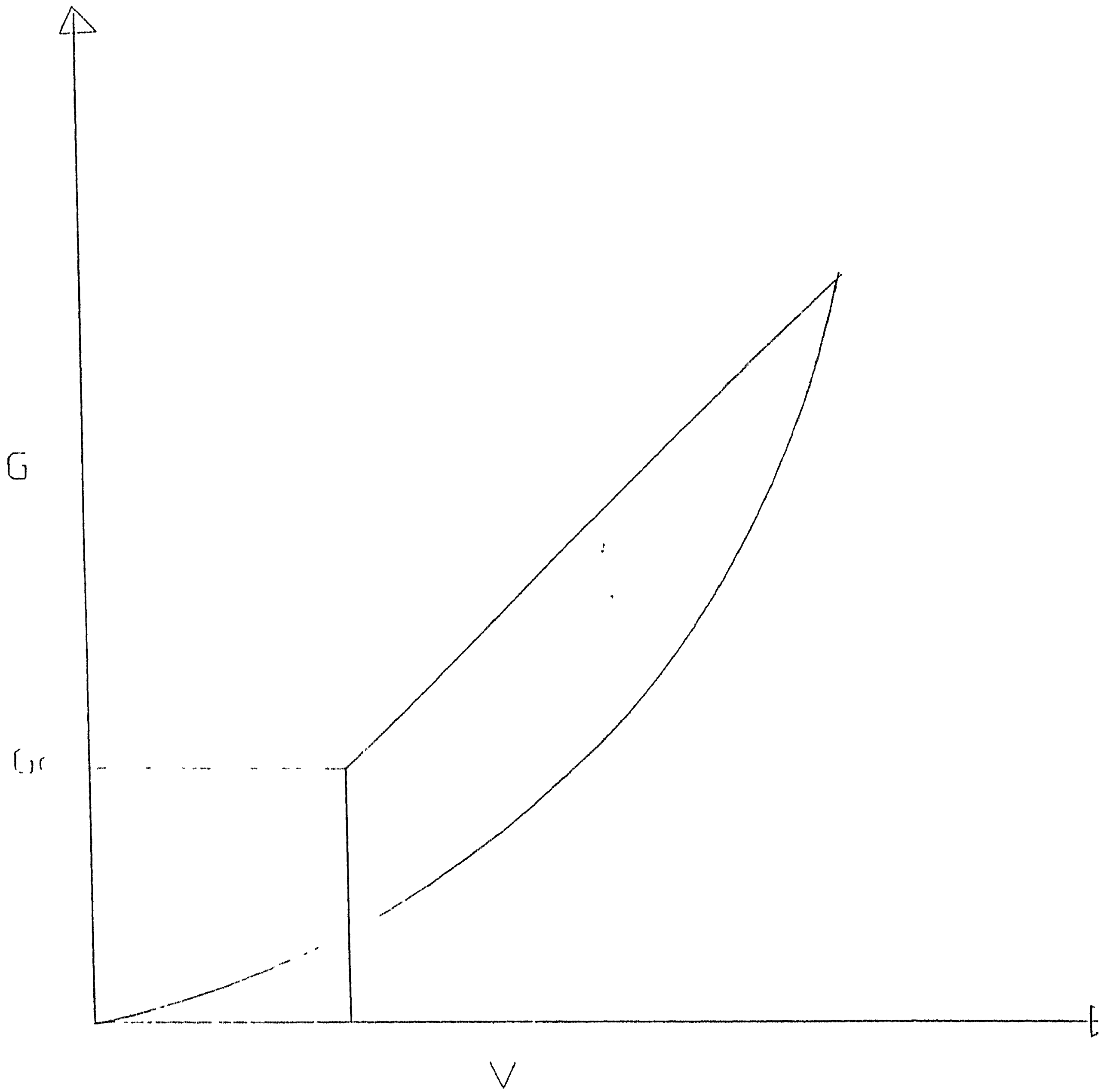

Fig. 3.2 Variation of G with propagation speed(hypothetical)

CHAPTER 4

RESULT AND DISCUSSION

This Chapter is divided into two parts: The first part consists of validation of computer code, while the second part consists of numerical results for different cases.

4.1 VALIDATION OF THE PRESENT METHOD

In order to validate the FEM code, a static problem of a plate with a crack in the centre subjected to uniform tension at edges parallel to the crack axis, is solved as shown in Fig. 4.1a. For the analysis only one fourth of the plate is considered (due to the symmetry) and the mesh used is shown in Fig. 4.1b.

The material properties and geometry are as follows :

Modulus of Elasticity	$E = 210$ GPa
Poissions ratio	$\upsilon = 0.3$
Density	$\rho = 7840$ Kg/m^3
Length of the Plate	$L = 0.104$ m
Width of the Plate	$W = 0.04$ m
Thickness of the Plate	$B = 0.024$ m
Crack Length	$2a = 0.024$ m
Virtual crack extension	$\Delta a = 0.001$ m
Far field stress	$\sigma_\infty = 5$ MPa

Theoretical value of energy release rate 4.084 J/m^2

Energy release rate using present analysis 3.967 J/m^2

Thus giving percentage in error = 2.8%

Thus, it can be observed that energy release rate obtained in the present analysis is good agreement with theoretical result (within 3%).

4.2 CRACK PROPAGATION IN QUASISTATIC CASE AND DETERMINATION OF C:

For this case a wedge loaded double cantilever beam specimen (DCB) is considered as shown in Fig.4.2a. Due to the symmetry, only the upper half of the specimen is modeled by finite elements and the mesh used is shown in Fig. 4.2b. The material properties and geometry are as follows:

Modulus of Elasticity	$E = 210$ Gpa
Poissions ratio	$\nu = 0.3$
Density	$\rho = 7840\ kg/m^3$
Length of specimen	$L = 0.06$ m
Height of specimen	$h = 0.027$ m
Thickness of specimen	$B = 0.024$ m
Crack length	$a = 0.04$ m
prescribed displacement	$u = 0.025$ mm

As explained in the sec. 3.2, the value of stiffness K_0 for the same configuration (geometry,bondary conditions,etc) has been obtained before the application of stiffness release model.

Fig. (4.2c) shows energy release rate due to the crack propagation with increase in crack length, for two different values of C. It is observed that using $C = 0$ energy release rate is constant within the element but no continuous from element to element. After a few trials using a value of -0.045 for C it is found that energy release rate is linearly decreasing within the element and continuous from element to element. Thus it can be seen that C value can be adjusted to the correct value easily.

To validate the energy release rate evaluation from the energy removed in the decrease of stiffness release J-integral value and energy release rate value are detemined for the same problem. Fig. 4.2d shows the comparison and it can be seen they match very well. From now onwards energy release rate G is used as it is easy to evaluate and far more general. A comparison of J-integral between force release model and stiffness release model as the crack propagates is shown in Fig. 4.2e with increase in crack length. It can be seen that J-integral obtained using stiffness release model is linearly decreasing within the element and is continuous from element to element and J-integral obtained using force release model is not linearly decreasing within the element. Thus, it can be seen that the stiffness release model gives gives far better results than force release model.

4.3 CRACK PROPAGATION AT CONSTANT SPEED IN DYNAMIC CASE

For this case a double cantilever beam (DCB) problem considered by Verma (1995) is analysed. Due to the symmetry, only upper half of the specimen is modeled by finite elements and mesh used is shown in Fig. 4.3. Verma (1995) experimentally measured crack length and velocity history which were used as the input data to calculate the energy release rate with respect to time. Material and geometry is same as mentioned in the previous section.

Fig. 4.4 presents the comparison of energy release rate due to crack propagation obtained using the present analysis and J-Integral calculated by Verma (1995) using force release model with respect to time. The crack propagartion speed measured was of the

value 1750 m/sec. which is about 0.6 C_s where C_s is shear wave speed.

The Fig. 4.4 shows the J-integral value as a function of time evaluated by Verma (1995) using force release model. It may also be observed the J value was zero until t = 25 μsec. Which was the time taken for the stress pulse to reach the crack tip. Fig. 4.4 also shows the energy release rate G calculated by present method. It can be observed that G evaluated once the crack starts propagating. It can be seen that J-integral value obtained by force release model becomes highly oscillatory and unstable after the crack propagates by few elements. On the other hand the present method gives smooth values without any oscillation. Another point worthnoting is force release model predicts J-integral value becomes zero after opening few elements. Whereas stiffness release model predicts a small value while the crack is propagating at high speed. Thus it may be infered present model predicts more accurate and consistent is the predictions.

Fig. 4.5, 4.6 and 4.7 shows the similar studies make on other experimental data. Comparision of G as a function of crack propagation speed can be seen in the following table.

Expt. No	Propagation Speed m/s	Propagation Toughness J/m^2
1	1750	41
2	1500	26
3	1350	40
4	1200	39

It appears the propagation toughness increases, but very slowly with crack propagation speed. Any stronger conclusion requires more data and more accurate measurements.

4.4 DESIGN PROBLEM - PREDICTION OF CRACK PROPAGATION:

The present method can be easily applied to predict the crack propagation history under the known loads, if crack propagation resistance of the material is known. That means that fracture resistance of the material, G_c as a function crack propagation velocity is supposed to be a known input data. Under a hypothetical data a limited study has been made, the geometry and loading are same as section 4.3. Under the assumption of crack initiation at time t = 40 μsec. further crack history is evaluated. Fig. 4.8 shows time vs. extension of crack and Fig. 4.9 shows propagation speed as a function of time. It should be noted that if the material data is fully available the initiation time also can be predicted.

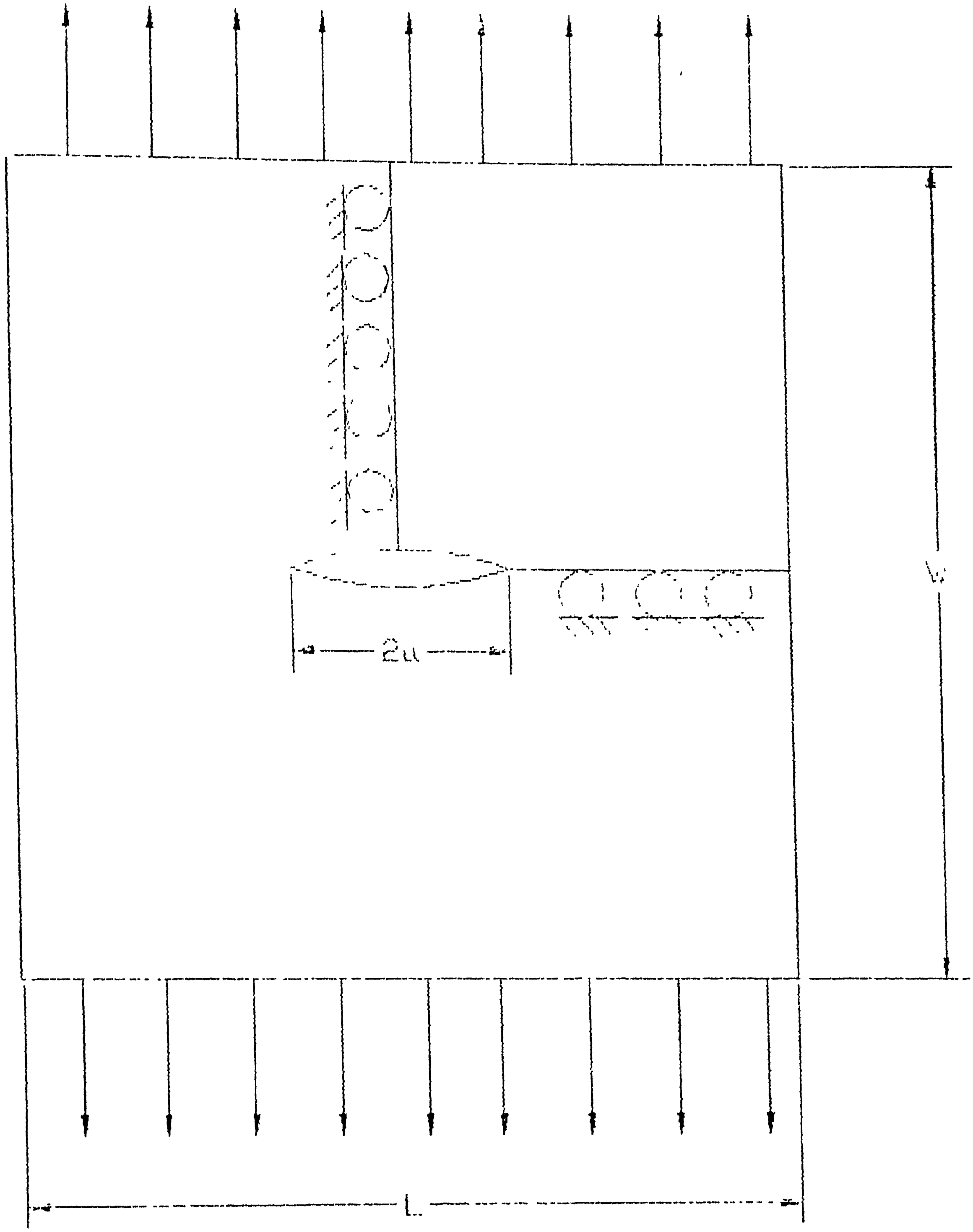

Fig. 4.1a Test problem of central crack

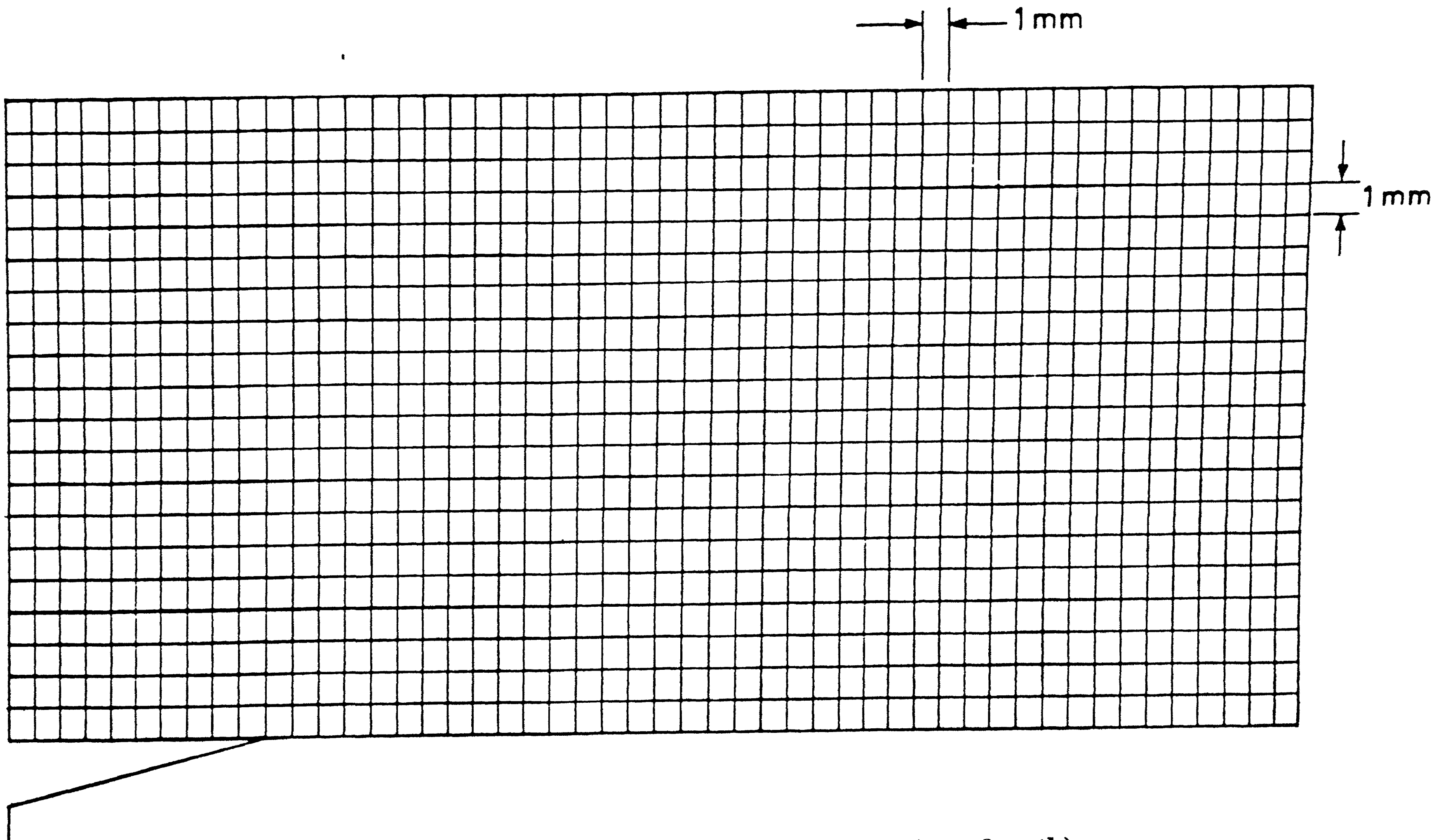

Fig. 4.1b Mesh for central crack problem (one fourth)

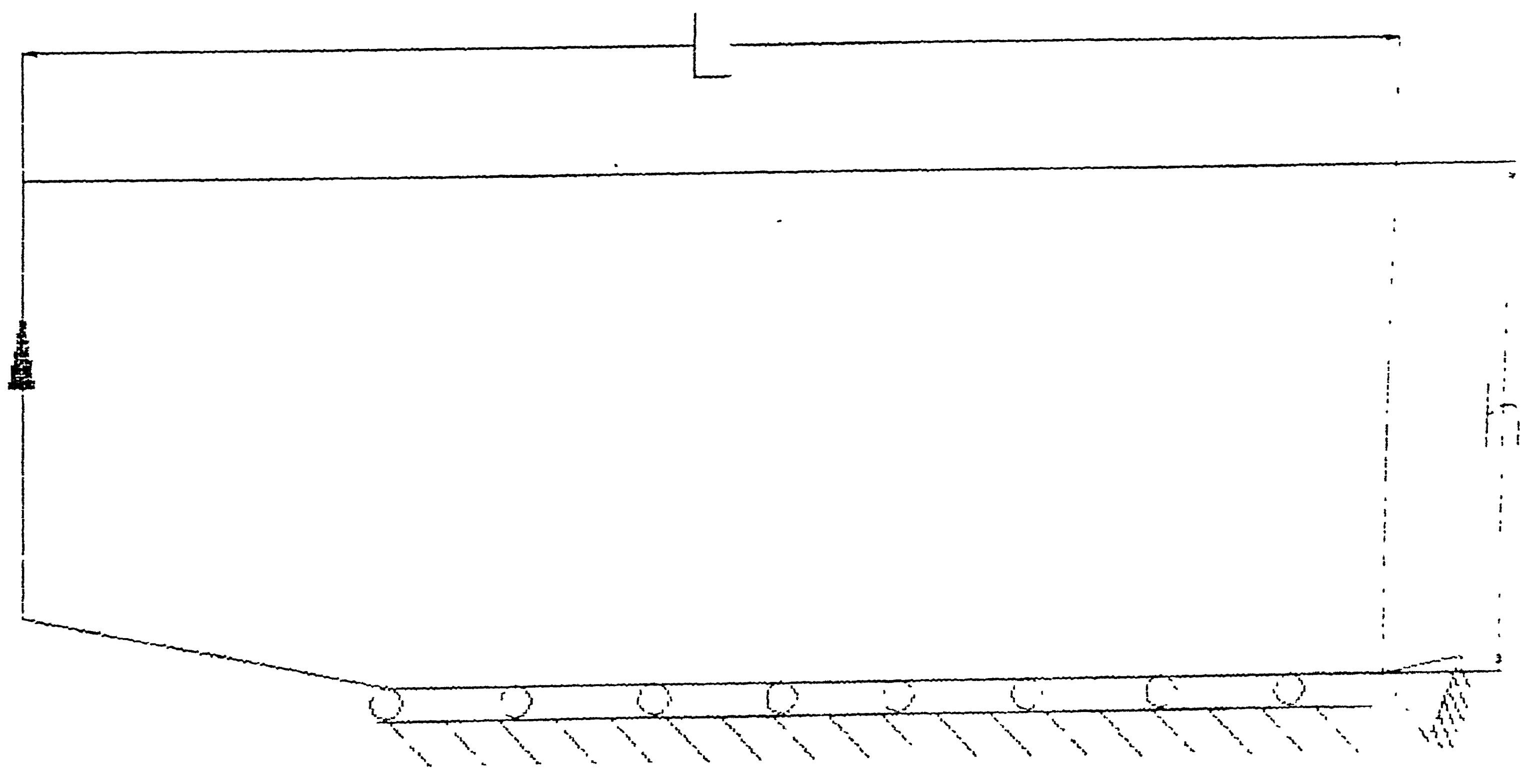

Fig. 4.2a DCB Specimen (one half)

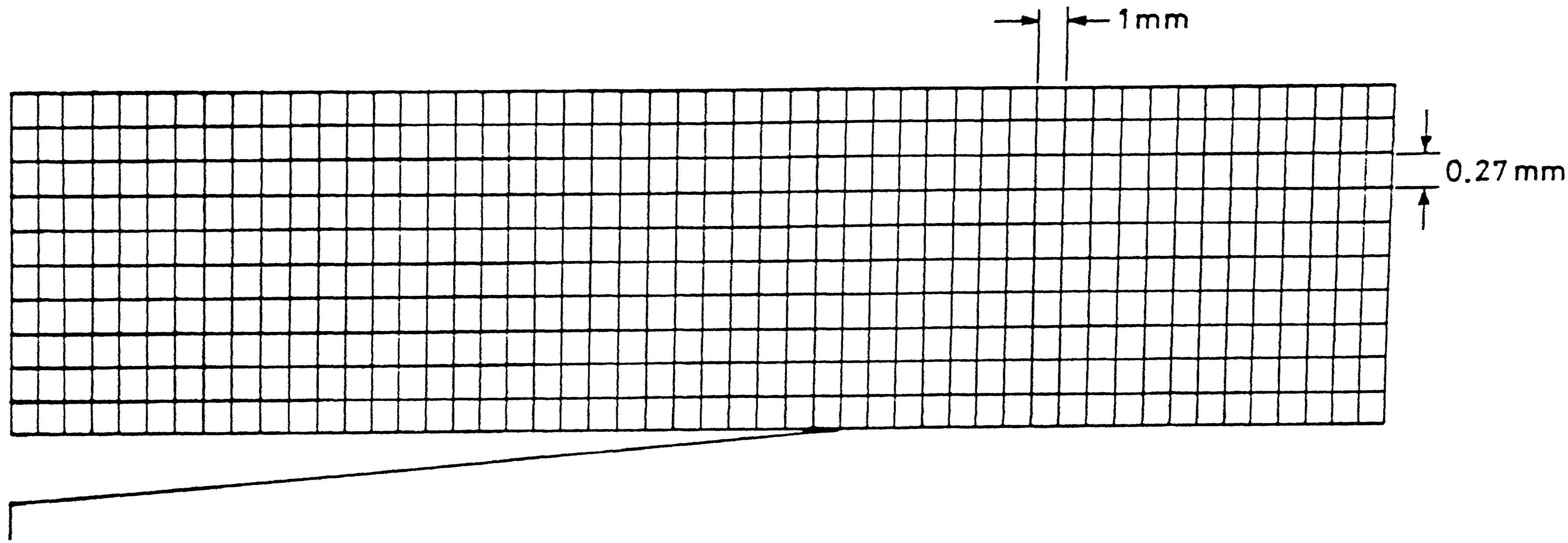

Fig. 4.2b Mesh(coarse) for DCB Specimen (one half)

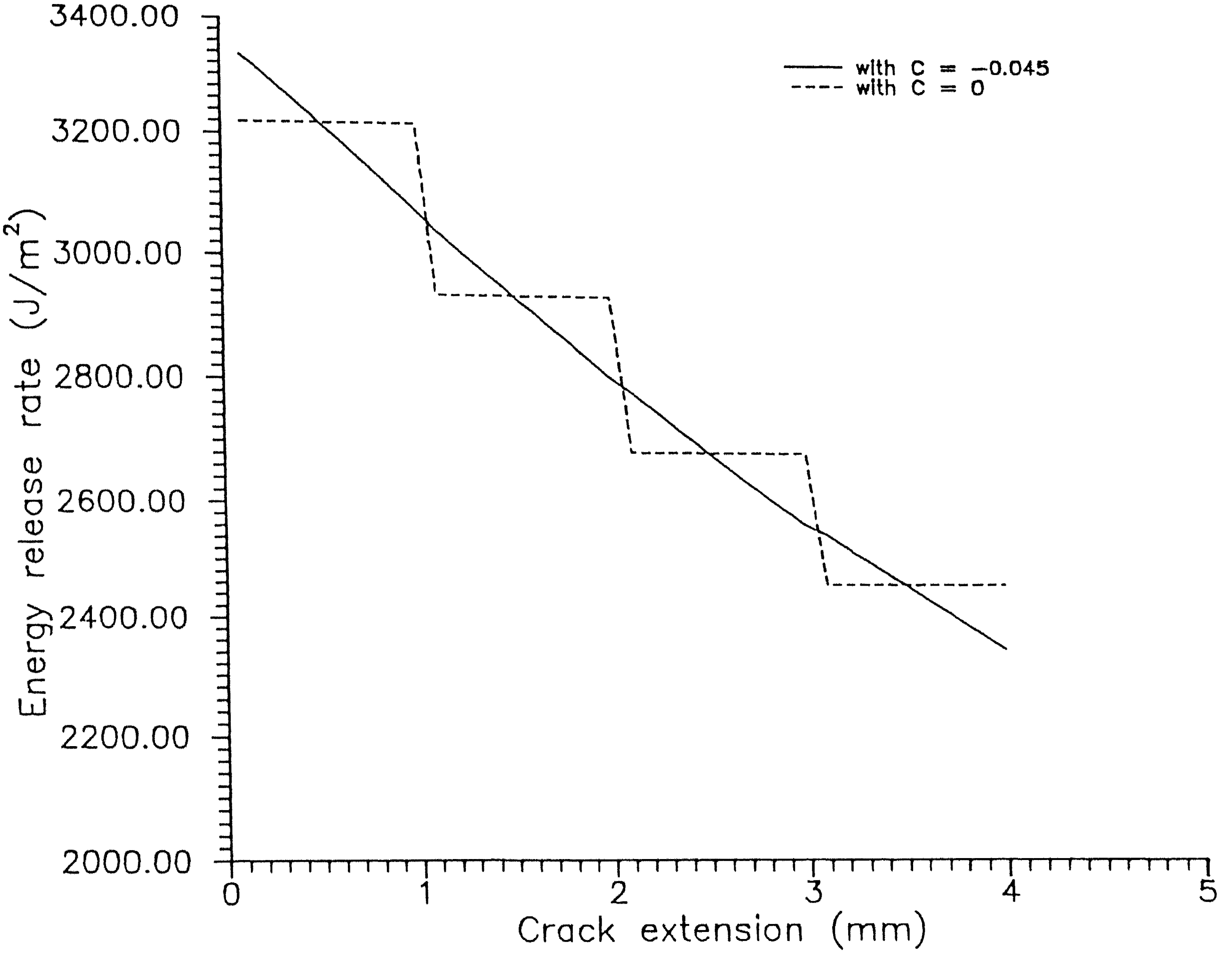

Fig. 4.2c Variation of G with increase in crack length

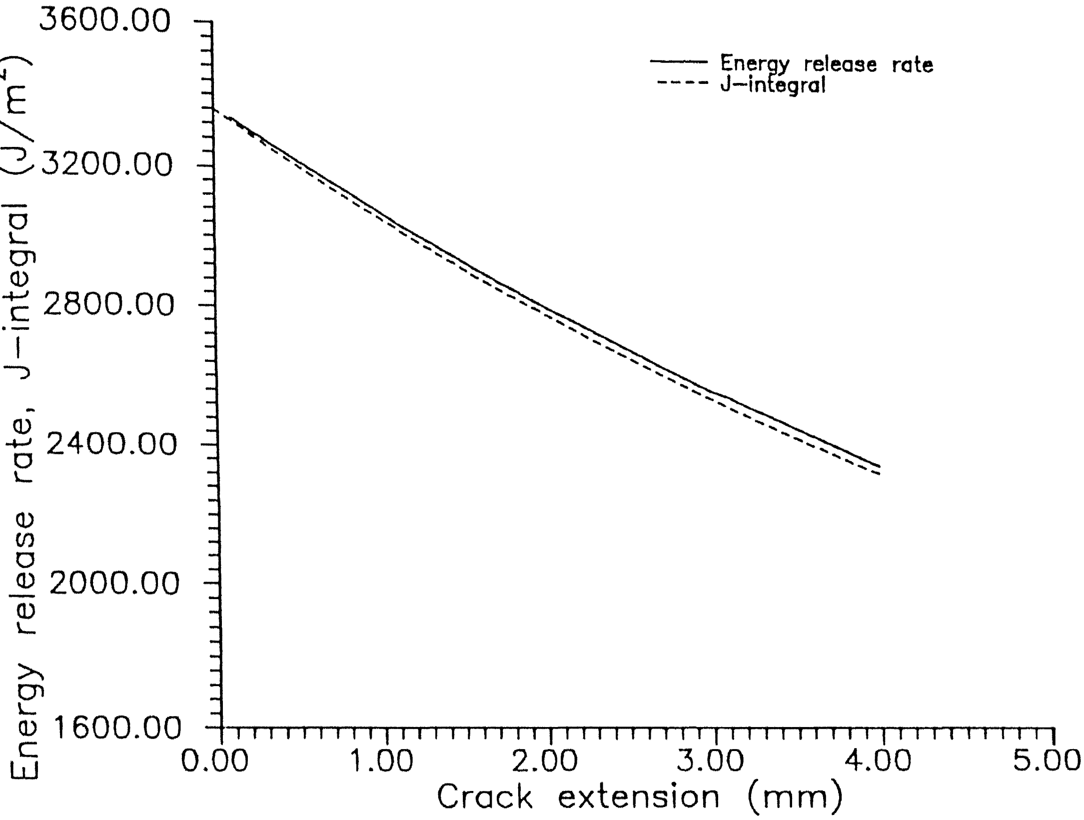

Fig. 4.2d Comparison of G and J with increase in crack length

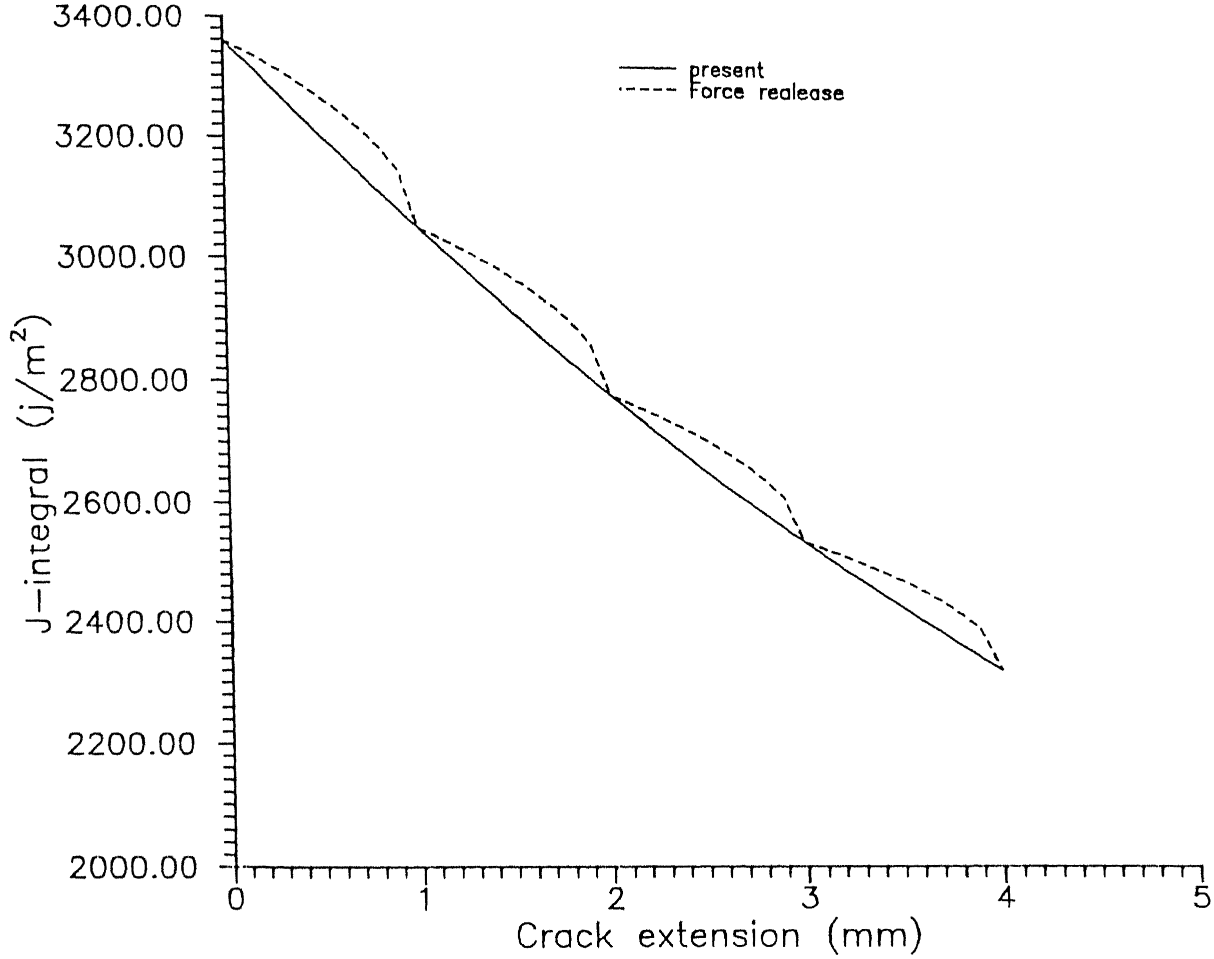

Fig. 4.2e Comparision of J between present model and force release model

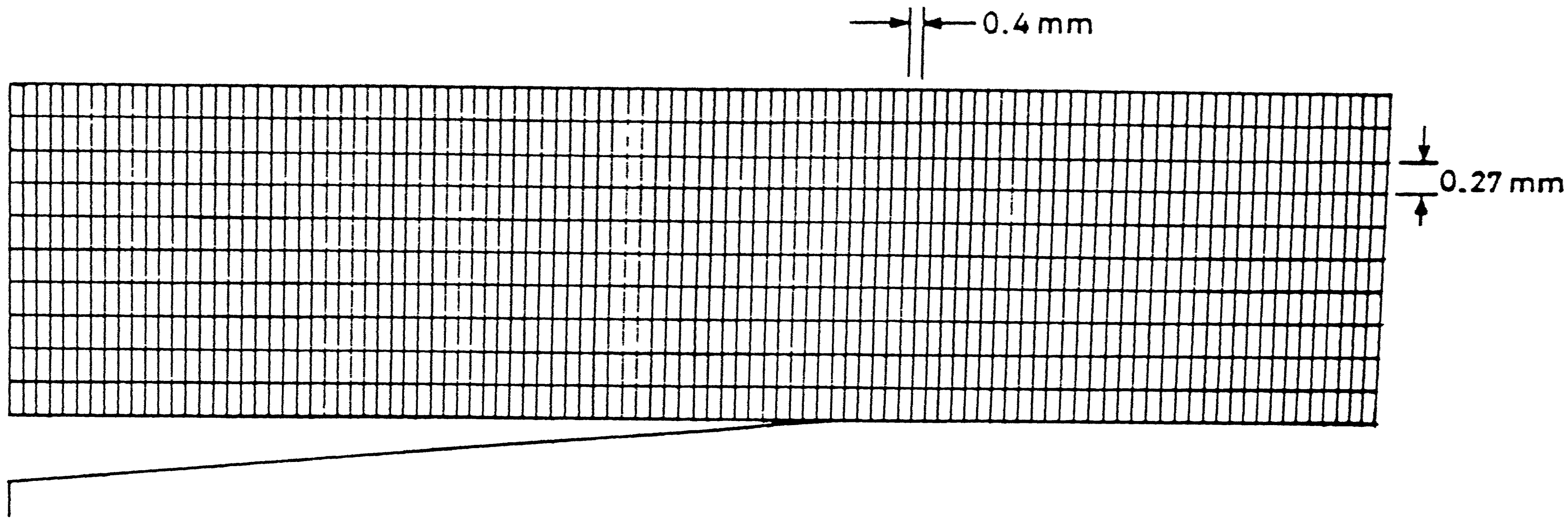

Fig. 4.3 Mesh(fine) for DCB specimen (one half)

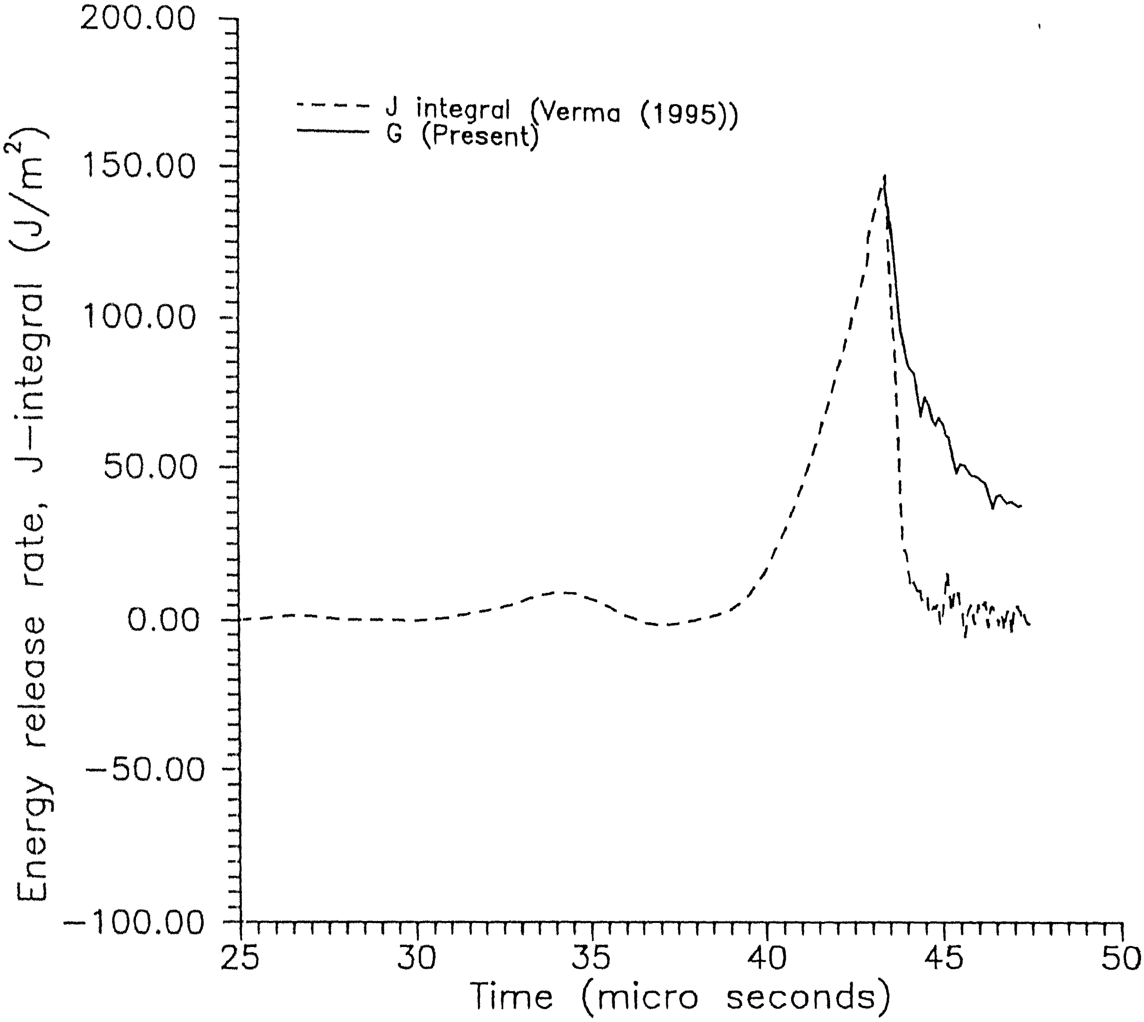

Fig. 4.4 Comparison of G and J with time for Expt.1

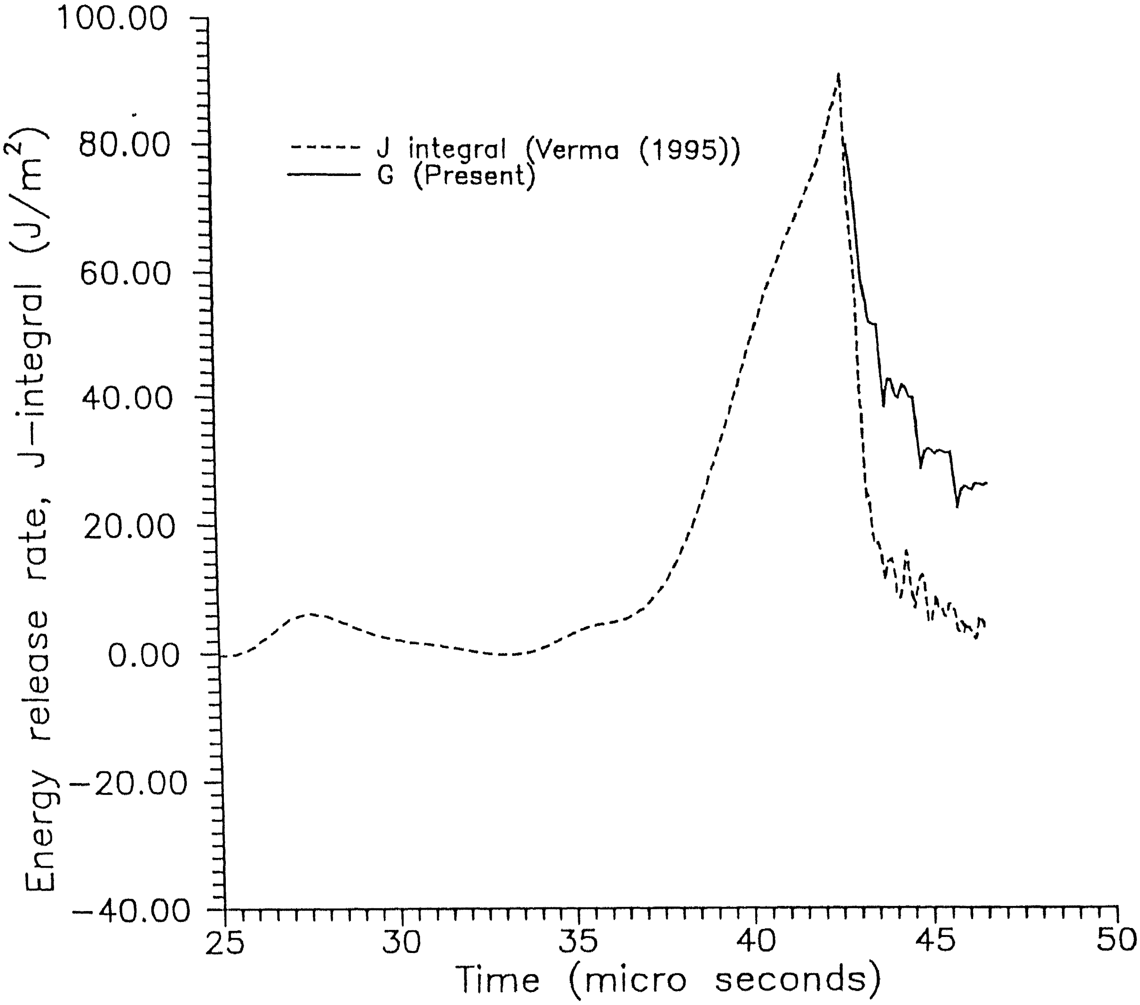

Fig. 4.5 Comparison of G and J with time for Expt. 2

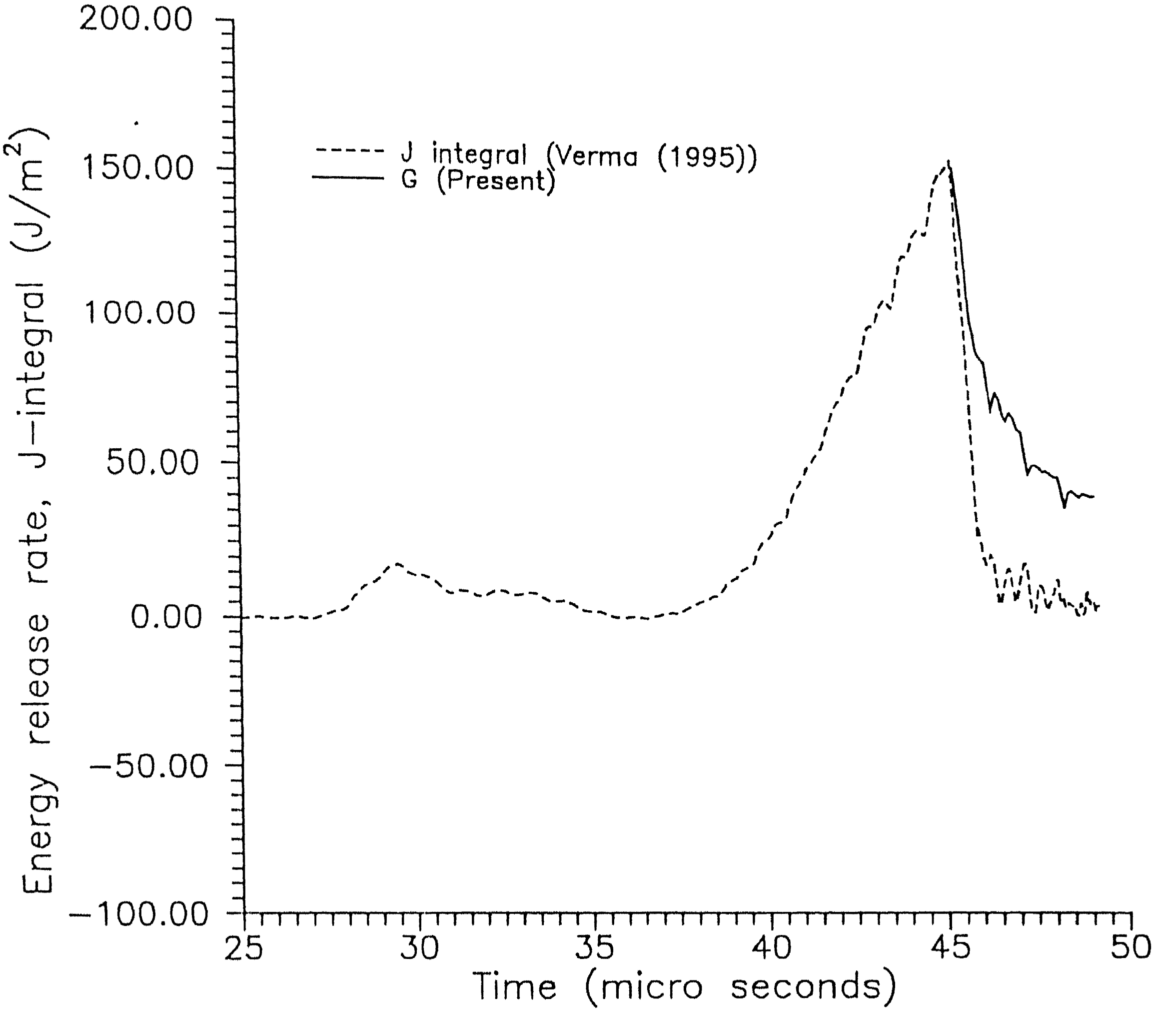

Fig. 4.6 Comparison of G and J with time for Expt. 3

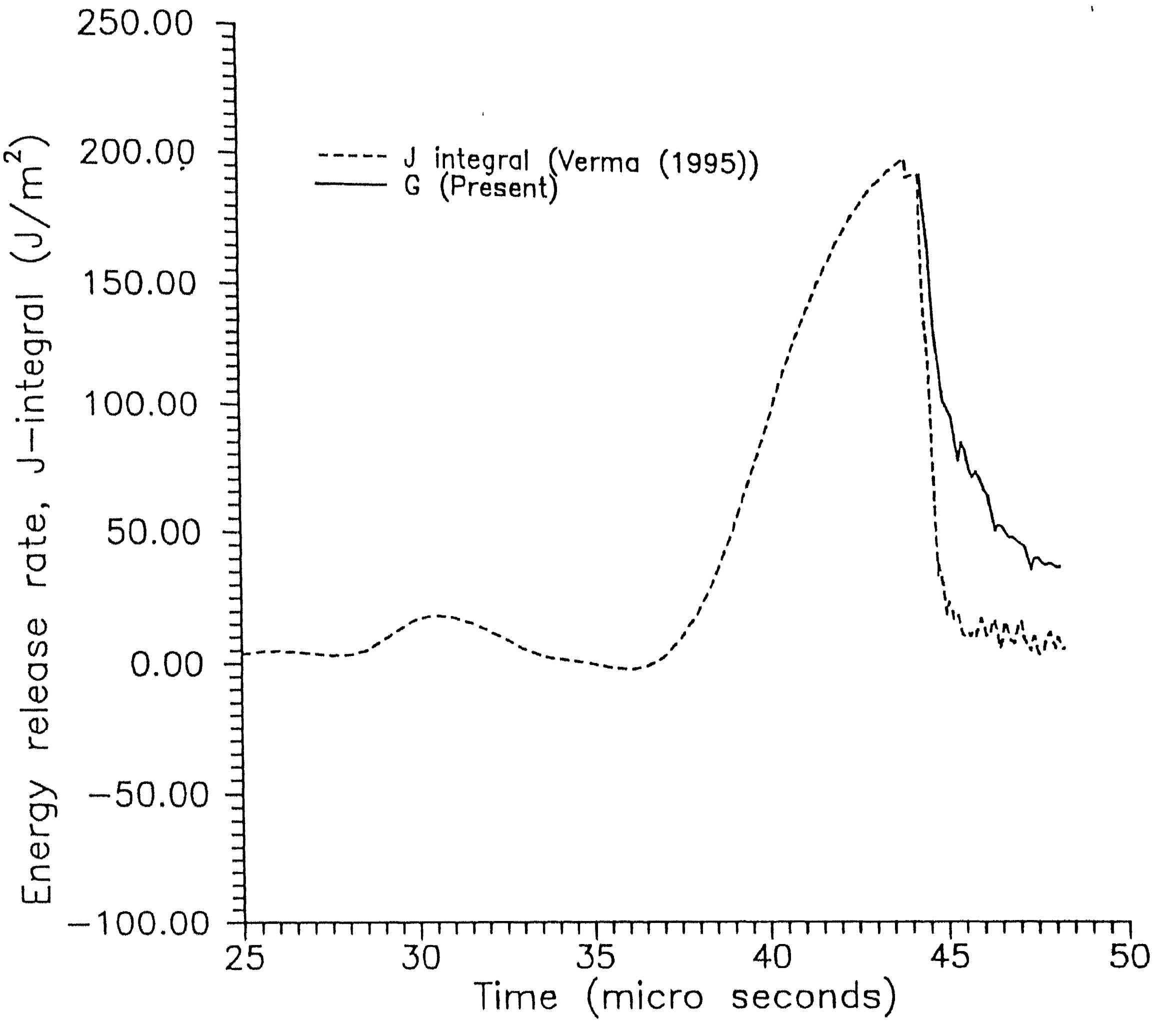

Fig. 4.7 Comparison of G and J with time for Expt. 4

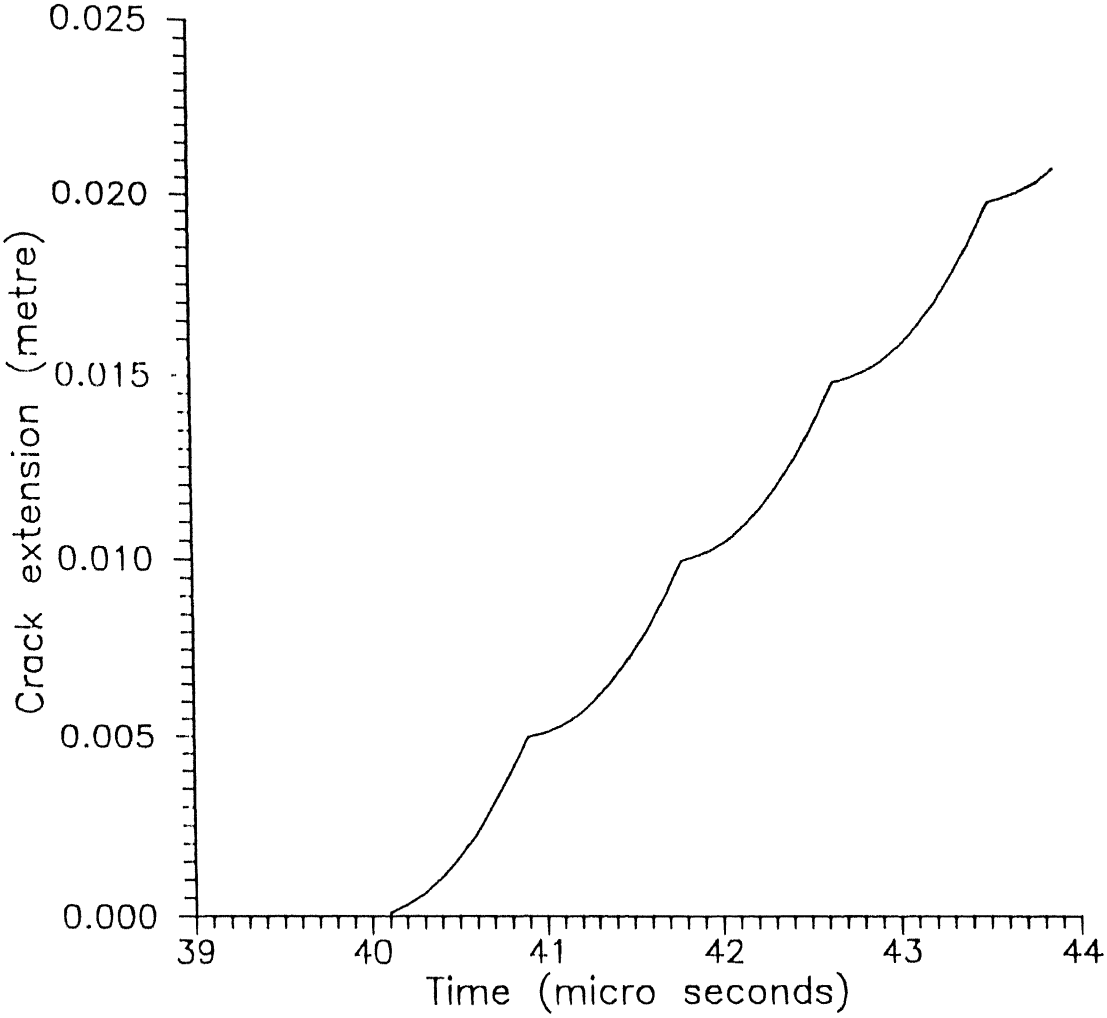

Fig. 4.8 Variation of crack extension with time

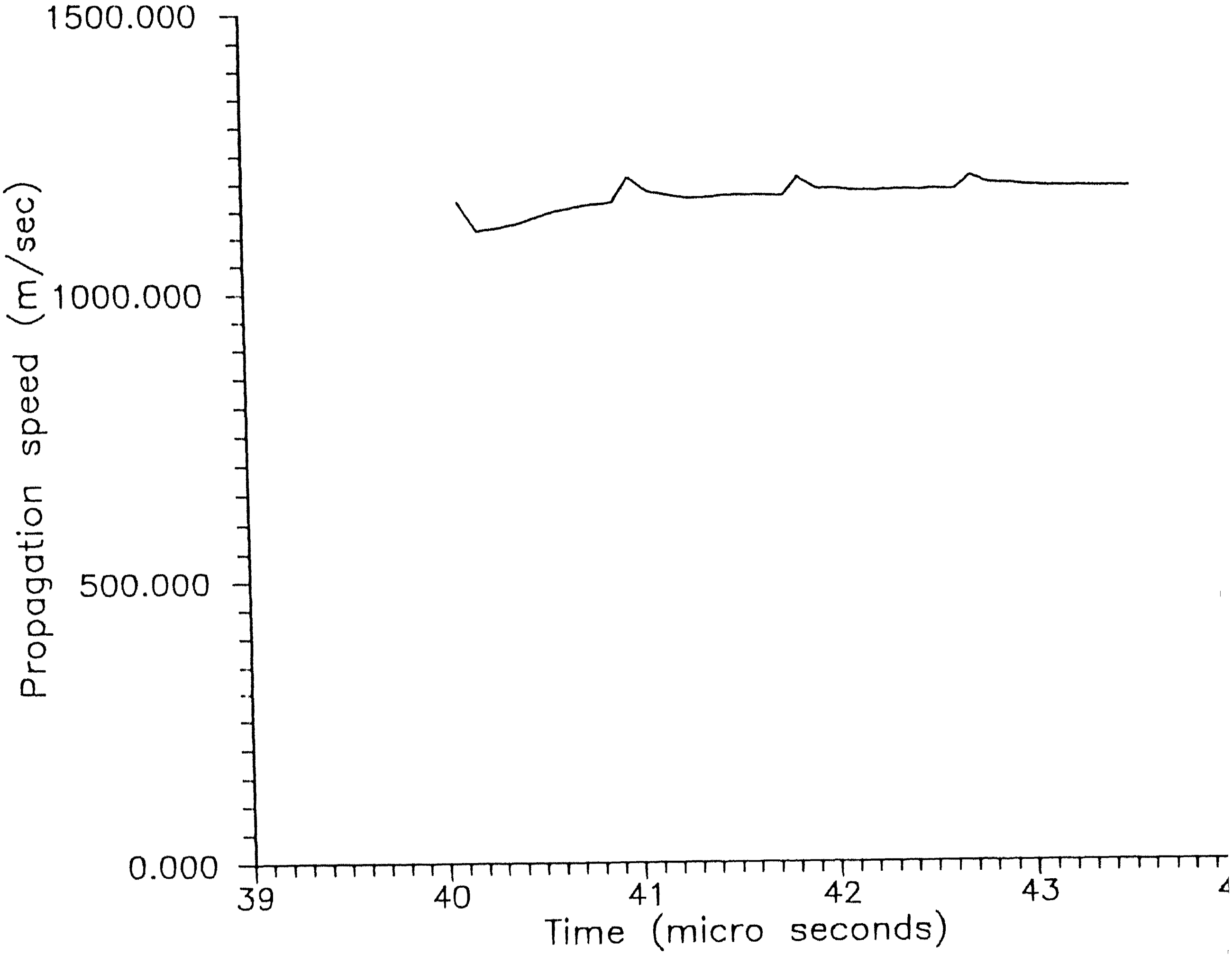

Fig. 4.9 Variation of Propagation speed with time

CONCLUSIONS AND SCOPE FOR FUTURE WORk

5.1 CONCLUSIONS:

The present work proposed a stiffness release model to investigate high speed crack propagation under dynamic conditions. Based on the results and discussions present in Chapter4 the following conclusion are drawn :

1. Stiffness release model offers non oscillatory and more stable solution.
2. It predicts a finite (though small) propagation toughness unlike force release model.
3. Design problems particularly those involving caack arrest can be easily solved.

5.2 SCOPE FOR FUTURE WORK:

1. Any strong conclusions on propagation toughness require more experimental data and accurate measurements.
2. The predetermination of K_0 value by quasitatic analysis may require some study so as to simplfy procedure.

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